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# GENERATION OF SIMULATED RADAR DATA FOR JOINT SURVEILLANCE SYSTEM

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This technical report has been reviewed and is approved for publication.

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20. Abstract (continued)

data generation algorithms; thus, the formulation is not well understood. This report derives the algorithm for calculating the range and azimuth associated with a simulated aircraft.

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#### 1.0 INTRODUCTION

The radar data generator calculates range and azimuth given the geographic coordinates of a radar, the simulated stereographic coordinates (X,Y) and height of an aircraft, and the geographic coordinates of the JSS region center. The generation of a radar report involves a coordinate transformation, a rotation and a conversion. The transformation is from stereographic coordinates into a rectangular coordinate system whose axes are directed north, east and vertical from the region center. The rotation maps the coordinates from this region centered rectangular coordinate system to a system whose axes are directed north, east and vertical at the radar site. The conversion from Cartesian to polar coordinates provides range and azimuth. This report derives the transformation and rotation formulary to be used for radar data generation in the JSS.

### Background

The algorithm used for generation of simulated radar data is in effect the inverse of the coordinate conversion and transformaused to project radar reports onto a common coordinate plane.

Radar reports from a network of sensors are typically converted to common plane coordinates by stereographic projection. The equations for determining locations on a stereographic plane were developed for a sphere because the mathematics were simpler than for an ellipsoidal earth model. The equations for stereographically

projecting a point referenced to the earth's surface reflect a two-step process: (a) points on the ellipsoid are conformally mapped onto a sphere, called the conformal sphere, and then (b) the mapped points on the conformal sphere are stereographically projected onto a plane tangent to the conformal sphere. The radius of the conformal sphere is calculated to minimize distortion in the projection process. This radius is shown in another document to very inversely with region size.

#### 2.0 COORDINATE TRANSFORMATION

Assume that one is given the position (X,Y) of an object stereographically projected on a plane tangent to a conformal sphere of radius  $E_0$ . The problem is to find the range and azimuth coordinates  $(\rho,\theta)$  of the object measured with respect to a radar on the surface of the earth.

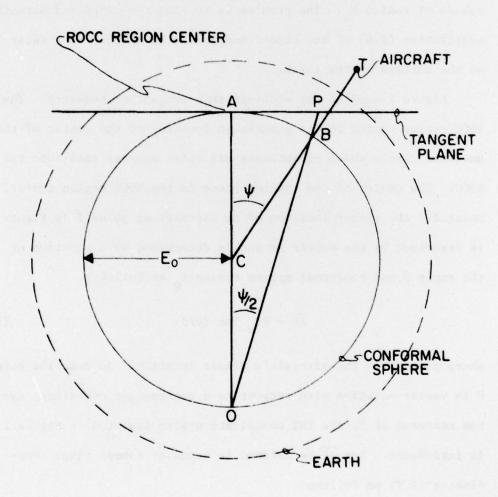
Figure 1 depicts the stereographic projection geometry. The ROCC region center is the geographic location of the center of the smallest circle which encompasses all radar sensors tied into the ROCC. The center of the tangent plane is the ROCC region center, point A. The planar location of an aircraft at point T in Figure 1 is described by the vector  $\overrightarrow{AP}$  and is determined as a function of the angle  $\psi$  and conformal sphere radius  $E_{o}$  as follows:

$$\overline{AP} = 2E_{O} \operatorname{Tan} (\psi/2)$$
 (1)

where point P is the aircraft's planar location. To describe point P in vector notation with respect to a rectangular coordinate system centered at 0, the ENZ coordinate system depicted in Figure 2 is introduced. Let  $\overline{AP}$  be defined in terms of common plane coordinates (X,Y) as follows:

$$\overline{AP} = X \overline{E} + Y \overline{N} + O\overline{Z}$$

$$\overline{AP} = \begin{bmatrix} X \\ Y \\ 0 \end{bmatrix}$$



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FIGURE 1
STEREOGRAPHIC PROJECTION GEOMETRY

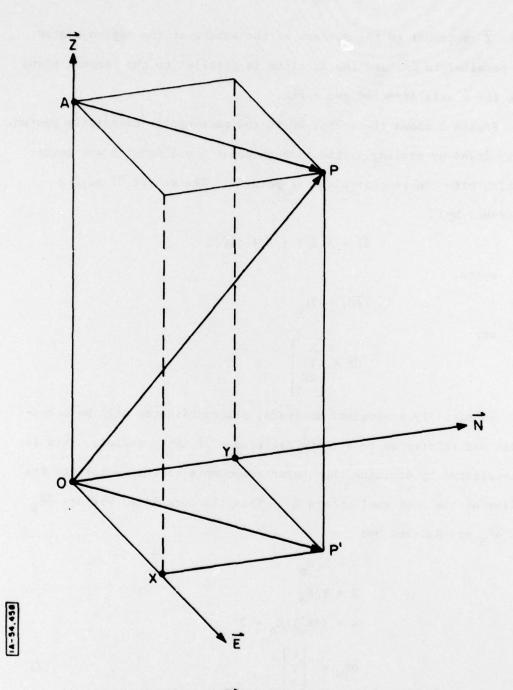


Figure 2 VECTOR OP IN RECTANGULAR COORDINATES

where  $\overline{Z}$  is normal to the surface of the sphere at the region center and parallel to  $\overline{OA}$ , and the EN plane is parallel to the tangent plane with the  $\overline{N}$  axis directed due north.

Figure 2 shows the vector  $\overline{OP}$  in the rectangular coordinate system.  $\overline{OP}$  is found by passing a line through point 0 and point B and intersecting with the tangent plane at point P. The vector  $\overline{OP}$  may be described by:

$$\overline{OP} = X \overline{E} + Y \overline{N} + |\overline{AO}|\overline{Z}$$

where:

$$|\overline{AO}| = 2E_{o}$$

or:

$$\overline{OP} = \begin{bmatrix} X \\ Y \\ 2E_0 \end{bmatrix}$$

To simplify subsequent analysis, all coordinates will be normalized and referenced to a conformal sphere of unity radius. This is accomplished by dividing the scalar components of the vectors by the radius of the conformal sphere  $\mathbf{E}_{_{\mathbf{O}}}$ . Thus the normalized vectors  $\overline{\mathrm{OP}}_{_{\mathbf{N}}}$  and  $\overline{\mathrm{AP}}_{_{\mathbf{N}}}$  are defined by:

$$U = X/E_{o}$$

$$V = Y/E_{o}$$

$$W = (2E_{o})/E_{o} = 2$$

$$\overline{OP}_{N} = \begin{bmatrix} U \\ V \\ 2 \end{bmatrix}$$
(2)

and:

$$\overline{AP}_{N} = \begin{bmatrix} U \\ V \\ 0 \end{bmatrix} \tag{3}$$

The vector  $\overline{\text{CT}}$  in Figure 1 is constructed from the center of the conformal sphere to an aircraft at point T. The magnitude of  $\overline{\text{CT}}$  is the sum of the aircraft height ( $\text{H}_{\overline{\text{T}}}$ ) and the radius of the earth as shown in Figure 3. Normalizing the vector  $\overline{\text{CT}}$  as described previously, the magnitude of  $|\overline{\text{CT}}_{N}|$  is as follows:

$$|\overline{CT}_{N}| = (E_{S} + H_{T})/E_{O}$$
 (4)

where:

Es = Earth radius,

 $H_{T}$  = Aircraft height, and

E = Conformal sphere radius

The vector  $\overline{\text{CT}}_{N}$  may be defined as:

$$\overline{CT}_{N} = |\overline{CT}_{N}|\overline{CB}_{N}$$

where  $\overline{\text{CB}}$  is a unit vector parallel to  $\overline{\text{CT}}$  and where:

$$\overline{CB}_{N} = \begin{bmatrix} c_{1} \\ c_{2} \\ c_{3}^{2} \end{bmatrix}$$
 (5)

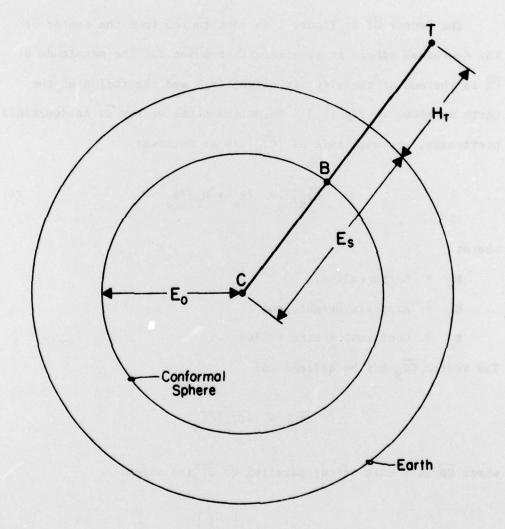


FIGURE 3\_ LENGTH OF CT

Let us now derive the scalar components  $C_1$ ,  $C_2$ , and  $C_3$  of the vector  $\overline{CB}_N$  in terms of  $\overline{CO}_N$  and  $\overline{OB}_N$  as shown in Figure 4:

Vectors  $\overline{OB}_N$  and  $\overline{OP}_N$  are in the same direction but with different magnitudes so  $\overline{OB}_N$  can be defined as:

$$\overline{OB}_{N} = K \overline{OP}_{N} = K \begin{bmatrix} U \\ V \\ 2 \end{bmatrix}$$

Thus:

$$K = \frac{|\overline{OB}_{N}|}{|\overline{OP}_{N}|}$$
 (6)

To solve for K we note that the triangles  $\Delta BAO$  and  $\Delta APO$  in Figure 4 are similar. The triangles share angle AOB, and angles OAP and OBA are both right angles; thus:

$$\frac{|\overline{OB}_{N}|}{|\overline{OA}_{N}|} = \frac{|\overline{OA}_{N}|}{|\overline{OP}_{N}|}$$
 (7)

From (6) and (7)

$$K = \frac{|\overline{OB}_{N}|}{|\overline{OP}_{N}|} = \frac{|\overline{OA}_{N}|^{2}}{|\overline{OP}_{N}|^{2}}$$

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FIGURE 4 SIMILAR TRIANGLES

By the law of right triangles:

$$|\overline{OP}_{N}|^{2} - |\overline{OA}_{N}|^{2} + |\overline{AP}_{N}|^{2}$$

80,

$$K = \frac{\left|\overline{OA}_{N}\right|^{2}}{\left|\overline{OA}_{N}\right|^{2} + \left|\overline{AP}_{N}\right|^{2}}$$
 (8)

From Equation 3:

$$|\overline{AP}_N|^2 = u^2 + v^2 + o^2$$

and vector  $\overline{OA}_N$  is the diameter of a unit radius circle so:

$$|\overline{OA}_{N}|^2 = 4$$

Thus, K is found to be:

$$K = \frac{4}{4 + u^2 + v^2} \tag{9}$$

From equation (6)  $\overline{OB}_{N}$  is defined as:

$$\overline{OB}_{N} - K \begin{bmatrix} U \\ V \\ 2 \end{bmatrix} - \begin{bmatrix} 4U/(4 + U_{2}^{2} + V_{2}^{2}) \\ 4V/(4 + U_{2}^{2} + V_{2}^{2}) \\ 8/(4 + U^{2} + V^{2}) \end{bmatrix}$$
(10)

From Figure 4, we note that:

$$\overline{CB}_N = \overline{OB}_N + \overline{CO}_N$$

where:

$$\overline{co}_{N} - \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

Thus,

$$\overline{CB}_{N} = \begin{bmatrix} 4U/(4 + U_{2}^{2} + V_{2}^{2}) \\ 4V/(4 + 2U + V_{2}^{2}) \\ (4 - U_{2}^{2} - V_{2}^{2})/(4 + U_{2}^{2} + V_{2}^{2}) \end{bmatrix}$$
(11)

### Coordinate Rotation

Having obtained the unit vector  $\overline{\operatorname{CB}}_N$  that is directed toward the aircraft in the normalized ENZ coordinate system, the slant range and azimuth  $(\rho,\theta)$  from a radar site to the aircraft can be calculated. The ENZ coordinate system is centered at the ROCC Region Center; its axes are directed toward east, north and normal to the surface of the sphere at the region center. The latitude and longitude of the region center are defined as  $(\lambda_0, \rho_0)$  respectively.

To calculate  $(\rho, \theta)$ , the ENZ coordinate system must be rotated into a system whose axes are directed east, north and normal to the surface of the sphere at the radar sites.  $(\lambda_{\rm S}, \emptyset_{\rm S})$  are defined as the latitude and longitude of the radar site.

Positive longitudes,  $\lambda$ , will be measured counterclockwise, or west, from the prime meridian,  $\lambda=0^{\circ}$  located in Greenwich, England. Positive latitudes,  $\beta$ , will be measured north from the equator,  $\beta=0$ .

Figure 5 depicts a set of rotations that align the ENZ axes toward east, north and normal to the surface of the sphere at the radar site.

In summary the sequence of rotations is:

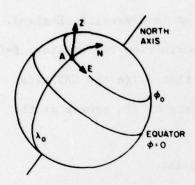
- 1) CCW about E, 0, to align N, with the earth's North axis
- 2) CCW about  $N_1$ ,  $\lambda_o$ , so that  $Z_2$  lies on the prime meridian
- 3) CW about N<sub>2</sub>,  $\lambda_s$ , so that Z<sub>3</sub> lies at longitude  $\lambda_s$
- 4) CW about  $E_3$ ,  $\rho_s$ , so that  $Z_4$  is vertical and  $N_1$  is directed north at the radar site.

The transformation equations to accomplish these rotations are as follows:

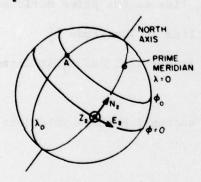
$$\begin{bmatrix} E_4 \\ N_4 \\ Z_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_s & -\sin\theta_s \\ 0 & \sin\theta_s & \cos\theta_s \end{bmatrix} \begin{bmatrix} \cos\lambda_s & 0 & -\sin\lambda_s \\ 0 & 1 & 0 \\ \sin\lambda_s & 0 & \cos\lambda_s \end{bmatrix}$$

$$\begin{bmatrix} \cos\lambda_o & 0 & \sin\lambda_o \\ 0 & 1 & 0 \\ -\sin\lambda_o & 0 & \cos\lambda_o \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_o & \sin\theta_o \\ 0 & -\sin\theta_o & \cos\theta_o \end{bmatrix} \begin{bmatrix} E \\ N \\ Z \end{bmatrix}$$

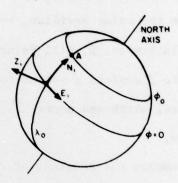
A) ORIGINAL (E,N,Z) COORDINATES



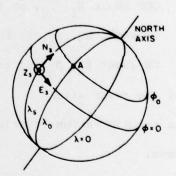
C) 2nd ROTATION: CCW ABOUT Npho Z2 LIES ON PRIME MERIDIAN



B) Ist ROTATION: CCW ABOUT E, φ. N. ALIGNED WITH NORTH AXIS OF EARTH



DI 3rd ROTATION: CW ABOUT N2. Ls Z3 LIES AT LONGITUDE A5



E) 4th ROTATION: CW ABOUT E3, €5

Z4 IS VERTICAL AND N4 IS DIRECTED NORTH AT THE RADAR

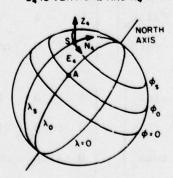


FIGURE 5
ROTATIONS TO RADAR SITE COORDINATES

To simplify the process into a product of 2 matrices instead of 4, let  $A(\theta_S, \lambda_S)$  equal the product of the first two matrices:

$$\mathbf{A}(\boldsymbol{\theta_{s}}, \boldsymbol{\lambda_{s}}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \boldsymbol{\theta_{s}} & -\sin \boldsymbol{\theta_{s}} \\ 0 & \sin \boldsymbol{\theta_{s}} & \cos \boldsymbol{\theta_{s}} \end{bmatrix} \begin{bmatrix} \cos \boldsymbol{\lambda_{s}} & 0 & -\sin \boldsymbol{\lambda_{s}} \\ 0 & 1 & 0 \\ \sin \boldsymbol{\lambda_{s}} & 0 & \cos \boldsymbol{\lambda_{s}} \end{bmatrix}$$

$$A(\theta_{s}, \lambda_{s}) = \begin{bmatrix} \cos \lambda_{s} & 0 & -\sin \lambda_{s} \\ -\sin \lambda_{s} \sin \theta_{s} & \cos \theta_{s} & -\sin \theta_{s} \cos \lambda_{s} \\ \cos \theta_{s} \sin \lambda_{s} & \sin \theta_{s} & \cos \theta_{s} \cos \lambda_{s} \end{bmatrix}$$

Let  $B(\mathbf{p}_0, \lambda_0)$  equal the product of the second two matrices:

$$B(\theta_{0},\lambda_{0}) = \begin{bmatrix} \cos \lambda_{0} & -\sin \lambda_{0} \sin \theta_{0} & \cos \theta_{0} \sin \lambda_{0} \\ 0 & \cos \theta_{0} & \sin \theta_{0} \\ -\sin \lambda_{0} & -\sin \theta_{0} \cos \lambda_{0} & \cos \theta_{0} \cos \lambda_{0} \end{bmatrix}$$

Notice that if we ignore the subscripts  $B(\emptyset, \lambda) = A^{T}(\emptyset, \lambda)$ .

The equation can then be written:

$$\begin{bmatrix} \mathbf{E}_{4} \\ \mathbf{N}_{4} \\ \mathbf{z}_{4} \end{bmatrix} - \mathbf{A}(\boldsymbol{\theta}_{s}, \lambda_{s}) \mathbf{A}^{T}(\boldsymbol{\theta}_{o}, \lambda_{o}) \begin{bmatrix} \mathbf{E} \\ \mathbf{N} \\ \mathbf{z} \end{bmatrix}$$

To transform the vector  $\overline{CB}_N$  into the  $(E_4, N_4, Z_4)$  coordinate system:

$$\overline{CB'}_{N} = A(\theta_{s}, \lambda_{s}) A^{T}(\theta_{o}, \lambda_{o}) \begin{bmatrix} 4U/(4 + U_{s}^{2} + V_{s}^{2}) \\ 4V/(4 + U_{s}^{2} + V_{s}^{2}) \\ (4-U^{2} - V^{2})/(4 + U^{2} + V^{2}) \end{bmatrix} = \begin{bmatrix} a_{1} \\ a_{2} \\ a_{3} \end{bmatrix}$$

The azimuth of the aircraft is calculated by:

$$\theta = ARCTAN\left(\frac{a_1}{a_2}\right)$$

To calculate slant range, the coordinate system must be rescaled, referenced to a sphere whose radius is that of the earth.

A vector from the center of the sphere to the radar is described as follows:

$$\overline{R} = \begin{bmatrix} 0 \\ 0 \\ (E_s + H_R) \end{bmatrix}$$

where:

E = Earth radius at the radar

 $H_{R}$  = Height of the radar antenna

A vector to the aircraft is found from CB'N by:

$$\overline{T} = (E_s + H_T) \overline{CB'}_N = (E_s + H_T) \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

where:

H<sub>T</sub> = Aircraft Height

Slant range ho is the difference between the R and T vectors:

$$\rho = |T - R|$$

$$\rho = \sqrt{\left[ (E_s + H_T)a_1 \right]^2 + \left[ (E_s + H_T)a_2 \right]^2 + \left[ (E_s + H_T)a_3 - (E_s + H_R) \right]^2}$$

## 3.0 CONCLUSION

This report explains a formulation used for simulated radar data generation. The algorithm would be absolutely accurate if the earth was a sphere. Slight errors may be introduced in conversion from an ellipsoidal to a spherical earth due to approximations in the conformal latitude calculation and the calculation of earth radius  $(E_s)$ . To minimize these errors, the conformal latitude and earth radius approximations should be as accurate as possible.